QUESTION 2006

Group – A
(Very Short Answer Type Questions)

1. Answers for any *ten* questions:

a) If $A=\{1, 2, 3\}$ and $B=\{x, y\}$, list all members of $A\times B$.

See Topic: SET THEORY, Long Answer Type Question No. 2(a).

b) If $A=\{2, 4, 6\}$ and $B=\{1, 3, 5, 7\}$, find $A\cap B$.

See Topic: SET THEORY, Long Answer Type Question No. 2(b).

c) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$.

See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 3.

d) If $x^3+2x^2+2ax+6$ is divisible by $x+2$, obtain a relation between *a* and *b*.

See Topic: EXPANSION OF FUNCTION, Short Answer Type Question No. 3.

e) Find $\lim_{x\to 3}\frac{x^2-9}{x-3}$.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 4.

f) If Z is the set of all integers and $f(x)=|x|$ as $x\in Z$. Show that f is not one to one.

See Topic: SET THEORY, Long Answer Type Question No. 2(c).

g) State fundamental theorem of Algebra.

See Topic: POLYNOMIAL, Short Answer Type Question No. 7.

h) If $A=\begin{pmatrix}1 & 0 & 2\\ 0 & -2 & 3\\ 1 & 0 & 0\end{pmatrix}$, find a matrix B so that $2A-B=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$.

See Topic: MATRICES, Short Answer Type Question No. 5.

i) If $y=4t$ and $x=\frac{1}{t}$, obtain value of $\frac{dy}{dx}$ at $t=-2$.

See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 5.

j) If α, β and γ are 3 roots of $x^3+qx+r=0$, find the value of $\alpha+\beta+\gamma$.

See Topic: POLYNOMIAL, Short Answer Type Question No. 8.

k) Show that the set of all real numbers does not form a group with respect to arithmetic multiplication.

See Topic: BINARY COMPOSITION, Short Answer Type Question No. 2.

l) If $\lim_{x\to a} f(x)=l(\neq 0)$, show that $\lim_{x\to a}|f(x)|=|l|$.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 5.

m) State Descartes rule of signs for number +ve roots of a polynomial equation with integral coefficients.

See Topic: POLYNOMIAL, Short Answer Type Question No. 9.

n) Obtain the remainder of $3x^4-x^3+2x^2-2x-1$ when it is divided by $x-3$.

See Topic: POLYNOMIAL, Short Answer Type Question No. 10.

Group – B

(Short Answer Type Questions)

2. When is a Group called commutative? Show that the set $G=\{1,-1,i,-i\}$ where $i^2=-1$ forms a commutative group under arithmetic multiplication.

See Topic: BINARY COMPOSITION, Long Answer Type Question No. 2.

3. Obtain a relation between p, q and r so that $x^3+px^2+qx+r=0$ has 3 roots that are in Arithmetic Progression.

See Topic: POLYNOMIAL, Long Answer Type Question No. 4.

4. Express $\begin{pmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{pmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

See Topic: MATRICES, Short Answer Type Question No. 6.

5. If $f(x)=3+2x$, if $=\frac{3}{2}\le x<0=3-2x$, if $0\le x<\frac{3}{2}=-3-2x$, if $x\ge\frac{3}{2}$. examine if $f(x)$ is continuous at $x=0$.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Long Answer Type Question No. 5.

6. Evaluate $\int\frac{dx}{(1+x)\sqrt{1-x^2}}$.

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 5.

Group – C

(Long Answer Type Questions)

7. a) If α,β and γ are roots of $x^3+qx+r=0$, find an equation whose roots are $\alpha^2\beta^2$, $\beta^2\gamma^2$ and $\gamma^2\alpha^2$.

b) If G is the set of all rationals except 1, show that G is commutative group under a composition $\circ$ given as $a\circ b=a+b-ab;\ a,b\in G$.

c) Evaluate $\lim_{x\to 0}\frac{\operatorname{cosec}x-\cot x}{x}$.

a) See Topic: POLYNOMIAL, Long Answer Type Question No. 5.

b) See Topic: BINARY COMPOSITION, Long Answer Type Question No. 3.

c) See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 6.

8. a) If $y = \sin\left(m\sin^{-1}x\right)$, then show that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

b) Evaluate $\int_0^{\pi/2} x^2 \sin x\,dx$.

c) If α, β and γ are 3 roots of $x^3 + px^2 + qx + r = 0$, obtain the value of $\Sigma(\alpha-\beta)^2$.

a) **See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 6.**

b) **See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 4.**

c) **See Topic: POLYNOMIAL, Long Answer Type Question No. 6.**

9. a) Find the inverse of $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$.

b) Expand $\log_e(1+x)$ in ascending powers of x with remainder after n terms in Lagrange's form.

c) Solve for x, y and z by Cramer's rule.

$$2x - y = 3$$
$$3y - 2z = 5$$
$$-x + 2z = -4.$$

a) **See Topic: MATRICES, Short Answer Type Question No. 7.**

b) **See Topic: EXPANSION OF FUNCTION, Long Answer Type Question No. 1.**

c) **See Topic: MATRICES, Long Answer Type Question No. 5.**

10. a) If by a rotation of rectangular co-ordinate axes without change of origin expression $ax+by$ and $cx+dy$ are transformed into $a'x'+b'y'$ and $c'x'+d'y'$, show that $a'd'-b'c' = ad-bc$.

See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 3.

~~b) If $f(x,y) = xy\dfrac{x^2-y^2}{x^2+y^2}$ ..... $= x$.~~

c) Evaluate $\int \frac{dx}{3+2\sin x}$.

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 6.

11. a) If origin in a rectangular co-ordinate system is shifted to $(3, -2)$ without changing directions of co-ordinate axes, obtain the transform of equation $3x + 4y = 5$.

b) Solve the following equations by matrix inversion method:

$$x + y + z = 4$$
$$2x - y + 3z = 1$$
$$3x + 2y - z = 1.$$

c) Show that collection of all 2×2 matrices of form $\begin{pmatrix} x & y \\ -y & \bar{x} \end{pmatrix}$, x and y being reals, form a ring with respect to usual matrix addition and multiplication.

a) See Topic: TRANSFORMATION OF CO-ORDINATES, Short Answer Type Question No. 3.

b) See Topic: MATRICES, Long Answer Type Question No. 6.

c) See Topic: BINARY COMPOSITION, Long Answer Type Question No. 4.